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Fatigue Damage Modeling for Prediction of Life– A Review

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Abstract—*This paper presents a review on the fatigue damage modeling for prediction of life. Various researchers have worked on the Fatigue damage analysis models and the prediction of life time due to fatigue loading. However, due to deficiencies in current life time prediction methodologies for these materials often require large factors of safety to be adopted. Therefore composite structures are often overdesigned and extensive prototype-testing is required to allow for an acceptable life time prediction. Improved damage accumulation models and life time prediction methodologies may result in a more efficient use of these materials and enhance the usage of these materials in critical areas like aerospace, marine and automobile industries. In general fatigue of fiber-reinforced composite materials is quite complex phenomenon, and a large research effort is being done today. Fiber-reinforced composites have a number of advantages as regards to life time in fatigue. The same does not apply to the number of cycles to initial damage nor to the evolution of damage. In this context, this paper brings out a review of the general considerations for fatigue damage modeling. Fatigue damage analysis and modeling can belong to these three categories; namely fatigue life models, which do not take into account the actual degradation mechanisms but use S-N curves or Goodman-type diagrams to assess the fatigue failure criterion; phenomenological models for residual stiffness/strength; and finally progressive damage models which use one or more damage variables like transverse matrix cracks, delamination size, etc,.[1]. This paper reviews the general considerations for fatigue damage analysis and the most important models proposed during the last decade.*

1. INTRODUCTION

Fiber reinforced composites are considered to have high specific stiffness and strength, therefore they are often selected for light weight structural applications. However, due to deficiencies in current life time prediction methodologies for these materials often require large factors of safety to be adopted. Therefore composite structures are often overdesigned and extensive prototype-testing is required to allow for an acceptable life time prediction. Improved damage accumulation models and life time prediction methodologies may result in a more efficient use of these materials and enhance the usage of these materials in critical areas like aerospace, marine and automobile industries.

2. FATIGUE DAMAGE MODELING GENERAL CONSIDERATIONS

In general fatigue of fiber-reinforced composite materials is quite complex phenomenon, and a large research effort is being done today. Fiber-reinforced composites have a number of advantages as regards to life time in fatigue. The same does not apply to the number of cycles to initial damage nor to the evolution of damage. Composite materials are inhomogeneous and anisotropic, and their behavior is more complicated than that of homogeneous and isotropic materials such as metals. The main reasons for this are the different types of damage that can occur (e.g. fiber fracture, matrix cracking, matrix crazing, fiber buckling, fiber-matrix interface failure, delaminations,...), their interactions and their different growth rates. Among the parameters that influence the fatigue performance of composites are: fiber type, matrix type, type of reinforcement structure (unidirectional, mat, fabric, braiding,...), laminate stacking sequence, environmental conditions (mainly temperature and moisture absorption), loading conditions (stress ratio R, cycle frequency,...) and boundary conditions. As a consequence the microstructural mechanisms of damage accumulation, of which there are several, occur sometimes independently and sometimes combined, and the predominance of one or other of them may be strongly affected by both material variables and testing conditions. There are a number of differences between the fatigue behavior of metals and fiber-reinforced composites. In metals, the stage of gradual and invisible deterioration spans nearly the complete life time. No significant reduction of stiffness is observed during the fatigue process. The final stage of the process starts with the formation of small cracks, which are the only form of macroscopically observable damage. Gradual growth and coalescence of these cracks quickly produce a large crack and final failure of the structural component. As the stiffness of a metal remains quasi unaffected, the linear relation between stress and strain remains valid, and the fatigue process can be simulated in most common cases by a linear elastic analysis and linear fracture mechanics. In a fiber-reinforced composite, damage starts very early and the extent of the damage zones grows steadily, while the damage type in these zones can change (e.g. small matrix cracks leading to large size delaminations). The gradual deterioration of a fiber-reinforced composite, with a loss of stiffness in the damaged zones, leads to a continuous redistribution of stress and a reduction of stress concentrations inside a structural component. As a consequence an appraisal of the actual state or a prediction of the final state (when and where final failure is to be expected) requires the simulation of the complete path of successive damage states. For this a number of parameters are often considered like applied stress, stress amplitude, loading frequency, modulus and material constants, environmental condition, etc.

According to Fong (1982) [2], there are two technical reasons why fatigue damage modeling in general is so difficult and expensive. The first reason is that, the several scales where damage mechanisms are present: from atomic level, through the sub-grain, grain and specimen levels, to the component and structural levels. The second reason is the impossibility of producing ‘identical’ specimens with well-characterized microstructural features.

Next, many models have been established for laminates with a particular stacking sequence and particular boundary conditions, under uniaxial cyclic loading with constant amplitude, at one particular frequency. The extrapolation to real structures with a stacking sequence varying from point to point, and more complex variations of the loads, is very complicated, if not impossible. Indeed some serious difficulties have to be overcome when fatigue life prediction of composite materials under general loading conditions is pursued: the governing damage mechanism is not the same for all states of stress level (Barnard et al (1985) [3], Daniel and Charewicz (1986) [4]. Failure patterns vary with cyclic stress level and even with number of cycles to failure,

- the load history is important. When block loading sequences are applied in low-high order or in high-low order, there can be a considerable difference in damage growth (Hwang and Han (1986a) [5],

- most experiments are performed in uniaxial stress conditions (e.g. uniaxial tension/compression), although these stress states are rather exceptional in real structures, - the residual strength and fatigue life of composite laminates have been observed to decrease more rapidly when the loading sequence is repeatedly changed after only a few loading cycles (Farrow (1989) [6]. This ‘cycle-mix effect’ shows that laminates that experience small cycle blocks, have reduced average fatigue lives as compared to laminates that are subjected to large cycle blocks, although the total number of cycles they have been subjected to, is the same for both laminates at the end of the experiment,

- the frequency can have a major impact on the fatigue life. Ellyin and Kujawski (1992) [7], investigated the frequency effect on the tensile fatigue performance of glass fiber-reinforced [± 45°] 5S laminates and concluded that there was a considerable influence of test loading frequency, especially for matrix dominated laminates and loading conditions, frequency becomes important because of the general sensitivity of the matrix to the loading rate and because of the internal heat generation and associated temperature rise.

Clearly a lot of research has still to be done in this domain. However several attempts have been made to extend models for uniaxial constant amplitude loading to more general loading conditions, such as block-type and spectrum loading and to take into account the effect of cycling frequency and multiaxial loads.

3. REVIEW OF THE EXISTING FATIGUE MODELS

This review aims to outline the most important fatigue models and life time prediction methodologies for fatigue testing of fiber-reinforced polymers. A rigorous classification is difficult, but a workable classification can be based on the classification of fatigue criteria by Sendeckyj (1990) [8]. According to Sendeckyj, fatigue criteria can be classified in four major categories: the macroscopic strength fatigue criteria, criteria based on residual strength, criteria based on residual stiffness, and the criteria based on the actual damage mechanisms. A similar classification has been used by the authors to classify the large number of existing fatigue models for composite laminates and consists of three major categories: fatigue life models, which do not take into account the actual degradation mechanisms but use S-N curves or Goodman-type diagrams and introduce some sort of fatigue failure criterion; phenomenological models for residual stiffness/strength; and finally progressive damage models which use one or more damage variables related to measurable manifestations of damage (transverse matrix cracks, delamination size) [1].

Although the fatigue behavior of fiber-reinforced composites is fundamentally different from the behavior exposed by metals, many models have been established which are based on the well-known S-N curves. These models make up the first class of ‘fatigue life models’. This approach requires extensive experimental work and does not take into account the actual damage mechanisms, such as matrix cracks and fiber fracture. The second class comprises the phenomenological models for residual stiffness and strength. These models propose an evolution law which describes the (gradual) deterioration of the stiffness or strength of the composite specimen in terms of macroscopically observable properties, as opposed to the third class of progressive damage models, where the evolution law is proposed in direct relation with specific damage [1]. Residual stiffness models account for the degradation of the elastic properties during fatigue. Stiffness can be measured frequently during fatigue experiments, and can be measured without further degrading the material (Highsmith and Reifsnider (1982) [9]). The model may be deterministic, in which a single-valued stiffness property is predicted, or statistical, in which predictions are for stiffness distributions. The other approach is based on the composite's strength. In many applications of composite materials it is important to know the residual strength of the composite structure, and as a consequence the residual life time during which the structure can bear the external load. Therefore the 'residual strength' models have been developed, which describe the deterioration of the initial strength during fatigue life. From their early use, strength-based models have generally been statistical in nature. Most commonly, two-parameter Weibull functions are used to describe the residual strength and probability of failure for a set of laminates after an arbitrary number of cycles. Since the damage mechanisms which govern the fatigue behavior of fiber-reinforced composites, have been studied intensively during the last decades, a last class of models have been proposed which describe the deterioration of the composite material in direct relation with specific damage (e.g. transverse matrix cracks, delamination size). These models correlate one or more properly chosen damage variables to some measure of the damage extent, quantitatively accounting for the progression of the actual damage mechanisms. These models are designated as 'mechanistic' models. Summarizing, the fatigue models can be generally classified in three categories: the fatigue life models; the phenomenological models for residual stiffness/strength; and the progressive damage models [1]. One of the important outcomes of all established fatigue models is the life time prediction. Each of the three categories uses its own criterion for determining final failure and as a consequence for the fatigue life of the composite component. The fatigue life models use the information from S-N curves or Goodman-type diagrams and introduce a fatigue failure criterion which determines the fatigue life of the composite specimen. Regarding the characterization of the S-N behavior of composite materials, Sendeckyj (1981) [10], advises to take into account three assumptions: the S-N behavior can be described by a deterministic equation, the static strengths are uniquely related to the fatigue lives and residual strengths at runout (termination of cyclic testing). An example of such a relationship is the commonly used 'strength-life equal rank assumption' (Hahn and Kim (1975) [11], Chou and Croman (1978)[12]) which states that for a given specimen its rank in static strength is equal to its rank in fatigue life, the static strength data can be described by a two-parameter Weibull distribution. Residual strength models have in fact an inherent 'natural failure criterion': failure occurs when the applied stress equals the residual strength (Harris (1985)[13], Schaff and Davidson (1997a)[14]). In the residual stiffness approach, fatigue failure is assumed to occur when the modulus has degraded to a critical level which has been defined by many investigators. Hahn and Kim (1976)[15] and O'Brien and Reifsnider (1981) [16], state that fatigue failure occurs when the fatigue secant modulus degrades to the secant modulus at the moment of failure in a static test. According to Hwang and Han (1986a)[17], fatigue failure occurs when the fatigue resultant strain reaches the static ultimate strain. Damage accumulation models and life time prediction methodologies are very often inherently related, since the fatigue life can be predicted by establishing a fatigue failure criterion which is imposed to the damage accumulation model. For specific damage types, the failure value of the damage variable(s) can be determined experimentally. Indeed some damage models are not applicable to notched specimens, because central holes and sharp notches at the edge of a specimen are known to be stress-concentrators. On the other hand such specimens are often used to deliberately initiate delaminations at a well-known site in the specimen. Although excellent review papers on the fatigue behavior of fiber-reinforced composites have been published in the past (Goetchius (1987)]18], Reifsnider (1990)[19], Stinchcomb and Bakis (1990)[20], Sendeckyj (1990)[8], Saunders and Clark (1993)[21]), this paper intends to focus on the existing modeling approaches for the fatigue behavior of fiber reinforced polymers. Since the vast majority of the fatigue models has been developed for and applied to a specific composite material and specific stacking sequence, it is very difficult to assess to which extent a particular model can be applied to another material type than the one it was tested for (glass/carbon fiber, thermoplastic/thermosetting matrix, unidirectional/woven/ stitched/braided reinforcement, unnotched/notched laminates,…), but this paper wants to give at least a comprehensive survey of the most important modeling strategies for fatigue behavior. Predicting fatigue life has been one of the most important problems in design engineering for reliability and quality. They have several practical uses: rapid design optimization during development phase of a product and predicting field use limits as well as failure analysis of product returned from the field or failed in qualification test. Fatigue analysis focus on the thermal and mechanical failure mechanism. Most fatigue failure can be attributed to thermo-mechanical stresses caused by differences in the coefficient of thermal and mechanical expansion. The fatigue failures will occur when the component experiences cyclic stresses and strains that produce permanent damage. There are two major components to fatigue failure: the initiation of fatigue cracks and the propagation of these cracks under cycling loading.

The fatigue life prediction consists of four primary steps. First a theoretical or constitutive equation, which forms the basis for modeling, is either defined or chosen. Appropriate assumptions need to be made in constructing the constitutive equation. Second, the constitutive equation is translated into Finite Element Analysis program and a model created. The Finite Element Analysis program calculates the predicted stress-strain values for the system under study and provides stress values for the simulated conditions. Third, the Finite Element Analysis results are used to create a model predicting the number of cycles to failure. Fourth, the model or results must be tested and verified by measurement data.

The main drawback of the fatigue life models is their dependency on large amounts of experimental input for each material, layup and loading condition (Schaff and Davidson

1997a)[14]. Moreover these models are difficult to extend towards more general loading conditions, where multiaxial stress conditions are imposed. On the other hand most of these models are straightforward to use and do not need detailed information about actual damage mechanisms.

When full-scale structural components are subjected to in-service fatigue loadings, stiffness can be a more adequate parameter as it can be measured non-destructively and the residual stiffness provides much less statistical scatter than residual strength (Highsmith and Reifsnider (1982)[9] , Hashin (1985)[22], Yang *et al* (1990, 1992)[23,24], Kedward and Beaumont (1992)[25], Whitworth (1998, 2000)[26,27]).

On the other hand, residual strength models possess a very natural failure criterion: if the residual strength falls to about the same level as the externally applied load, the material will fail (Harris (1985)[13]). Residual stiffness models are dealing with different definitions of 'failure' and already in the early 70s, Salkind (1972)[28], suggested to draw a family of S-N curves, being contours of a specified percentage of stiffness loss, to present fatigue data, quoting from Talreja (2000)[29], at the Second International Conference on Fatigue of Composites (June 2000).

4. FATIGUE LIFE MODELS

The first category contains the 'fatigue life' models: these models extract information from the S-N curves or Goodman-type diagrams and propose a fatigue failure criterion. They do not take into account damage accumulation, but predict the number of cycles, at which fatigue failure occurs under fixed loading conditions [1].

One of the first fatigue failure criteria was proposed by Hashin and Rotem (1973)[30]. They distinguished a fiber- failure and a matrix-failure mode.

Ellyin and El-Kadi (1990)[31], demonstrated that the strain energy density can be used in a fatigue failure criterion for fiber-reinforced materials. The fatigue life N_f was related to the total energy input W^t through a power law type relation.

Reifsnider and Gao (1991)[32], established a fatigue failure criterion, based upon an average stress formulation of composite materials derived from the Mori-Tanaka method (a method to calculate the average stress fields in inhomogeneities and their surrounding matrix). The criterion is at the micromechanics level and takes into account the properties of the constituents and the interfacial bond.

Fawaz and Ellyin (1994)[33], proposed a semi-log linear relationship between applied cyclic stress S and the number of cycles to failure N.

Harris and his co-workers (Harris (1985)[13], Adam *et al* (1994)[34], Gathercole *et al* (1994)[35]) who have performed extensive research on fatigue in composite materials, proposed a normalized constant-life model that expresses which combinations of mean and peak stress amplitudes give rise to the same number of cycles to failure.

Philippidis and Vassilopoulos (1999)[36], proposed a multiaxial fatigue failure criterion, which is very similar to the well known Tsai-Wu quadratic failure criterion for static loading.

Bond (1999)[37], has developed a semi-empirical fatigue life prediction methodology for variable-amplitude loading of glass fiber-reinforced composites.

Xiao (1999)[38], has modeled the load frequency effect for thermoplastic carbon/PEEK composites. Fatigue life prediction for 5 Hz and 10 Hz was based on the S-N data at 1 Hz. The reference S-N curve was modeled by a four-parameter power law relation.

Miyano *et al* (1994, 2000)[39,40]], developed a model for predicting tensile fatigue life of unidirectional carbon fiber-reinforced composites. The method is based on four hypotheses: (i) same failure mechanisms for constant strain-rate loading, creep and fatigue failure, (ii) same time-temperature superposition principle for all failure strengths, (iii) linear cumulative damage law for monotonic loading, and (iv) linear dependence of fatigue strength upon stress ratio.

Van Paepegem and Degrieck (2000b, 2001)[41,42], have implemented the model of Sidoroff and Subagio[43], into a commercial finite element code.

Hansen (1997,1999)[44,45], developed a fatigue damage model for impact-damaged woven fabric laminates, subjected to tension-tension fatigue.

Brondsted *et al* (1997a,1997b)[46,47], extended stiffness reduction to the life time prediction of glass fiber-reinforced composites. The predictions are based on experimental observations from wind turbine materials subjected to constant amplitude loading, variable amplitude block loading and stochastic spectrum loading.

Two types of residual strength models can be distinguished: the *sudden death* model and the *wear out* model. When composite specimens are subjected to a high level state of stress (low-cycle fatigue), the residual strength as a function of number of cycles is initially nearly constant and it decreases drastically when the number of cycles to failure is being reached. The *sudden death* model (Chou and Croman (1978,1979)[12,48]) is a suitable technique to describe this behavior and is especially used for high-strength unidirectional composites.

However at lower level states of stress, the residual strength of the laminate, as a function of number of cycles, degrades more gradually. This behavior is described by degradation models which are often referred to as *wearout* models.

In the *wearout* model, which was initially presented by Halpin *et al* (1973)[49], it is assumed that the residual strength R(n) is a monotonically decreasing function of the number of cycles n, and that the change of the residual strength can be approximated by a power-law growth equation.

Extensive experimental and theoretical research has been done by Schaff and Davidson (1997a, 1997b)[14,50]. They presented a strength-based wearout model for predicting the residual strength and life of composite structures subjected to spectrum fatigue loading.

Caprino and D'Amore (1998)[51], stressed the fact that a reliable model should reflect both the influence of the stress ratio R and the different fatigue behavior at low- and high-cycle fatigue.

Progressive damage models differ from the above mentioned models, in that they introduce one or more properly chosen damage variables which describe the deterioration of the composite component. These models are based on a physically sound modeling of the underlying damage mechanisms, which lead to the macroscopically observable degradation of the mechanical properties. The models have been subdivided into two classes: the damage models which predict the damage growth as such (e.g. number of transverse matrix cracks per unit length, size of the delaminated area), and the models which correlate the damage growth with the residual mechanical properties (stiffness/strength).

Owen and Bishop (1974)[52], were among the first researchers to investigate a wide range of glass fiber-reinforced composites. They tried to predict the initiation of damage at central holes in the specimens under static and fatigue loading.

Xiao and Bathias (1994a,1994b)[53,54], studied notched and unnotched woven glass/epoxy laminates with a strong unbalanced character: the mechanical properties in the 'warp' direction were much higher than those in the 'weft' direction. Although they did not propose a fatigue evolution law, they introduced fatigue ratios to compare the experimental data. The results showed that the unnotched and notched laminates have the same ratios of the fatigue strength to the ultimate tensile strength and that the fatigue strength ratios of notched and unnotched laminates for the three stacking sequences considered, are respectively equal to their respective static strength ratios. They also reported that the stacking sequence influences the fatigue life: when 90□ layers are constrained by 0□ layers, the damage in the 90□ layers cannot easily cross the interface between the 90□ plies and the other plies. As a consequence the damage trace is very sinuous through the thickness.

Bucinell (1998)[55], developed a stochastic model for the growth of free edge delaminations in composite laminates. The experiments were conducted for the stacking sequence [□45°/90°/0°]$_S$ of AS4/3501-6 coupons, where the location of the free edge delamination was observed to appear always in the 45°/90° interface. The growth model was derived from fracture mechanics principles.

This category of progressive damage models relates the damage variable(s) with the residual mechanical properties (stiffness/strength) of the laminate. The damage growth rate equations are often based on damage mechanics, thermodynamics, micromechanical failure criteria or specific damage characteristics (crack spacing, delamination area,…).

One of the first methods to calculate stiffness reduction due to matrix cracking is the shear-lag model, established by Highsmith and Reifsnider (1982)[9].

More recent models to calculate stiffness reduction due to matrix cracking are based on a variational approach, finite element analysis,… References can be found in Nuismer and Tan (1988)[56], Brillaud and El Mahi (1991)[57], El Mahi *et al* (1995)[58], Joffe and Varna (1999)[59], Pradhan *et al* (1999)[60], Smith and Ogin (1999)[61] and Kashtalyan and Soutis (2000)[62].

Talreja (1986, 1990)[63,64], presented a continuum damage model, where internal damage variables are characterized by vectorial/tensorial quantities. To determine the mechanical response in the presence of damage, stiffness-damage relationships are derived from a theory with internal variables based on thermodynamic principles, wherein the damage vectors/tensors have been taken as the internal state variables.

Bonora *et al* (1993)[65], have presented a semi-empirical model for predicting the mechanical properties degradation of a composite laminate due to transverse matrix cracks. The constitutive equations are based on the damage model of Talreja (1986, 1990)[63,64]. The damage variable D is a product of three parameters, related to the crack density, length and width.

Shokrieh (1996)[66], and Shokrieh and Lessard (1997a,1997b, 1998, 2000a, 2000b) [67-71] proposed a 'generalized residual material property degradation model' for unidirectionally reinforced laminates. In this model three approaches are combined: (i) polynomial fatigue failure criteria are determined for each damage mode, (ii) a master curve for residual strength/stiffness is established, and (iii) the influence of arbitrary stress ratio is taken into account by use of the normalized constant-life diagram developed by Harris (1985)[13].

Finally the fatigue life of a unidirectional ply under arbitrary state of stress and stress ratio is calculated using the normalized constant-life model developed by Harris (1985)[13].

Residual stiffness models describe the degradation of elastic properties during fatigue loading. The damage variable D is defined by a one-dimensional relation,

$D = 1 - E/E_0$, where E_0 is the undamaged modulus [9].

Hwang and Han (1986a)[17], proposed three cumulative damage models based on the fatigue modulus F(n) and the resultant strain.

The model of Sidoroff and Subagio[43], has been adopted very recently by other researchers, but often in terms of stress amplitude instead of strain amplitude.

5. CONCLUSIONS

Extensive research on fatigue modeling of fiber-reinforced composite materials has been done during the last decades. A lot of models have been proposed to predict damage accumulation and fatigue life for composites with various stacking sequences and fiber- and matrix-types under loading conditions that vary from constant amplitude loading to spectrum loading. Nevertheless research in this domain should be addressed further attention, in order to meet the challenge of developing models with a more generalized applicability in terms of loading conditions and the materials used. While the residual strength is a meaningful measure of fatigue damage, it does not allow for non-destructive evaluation as such. It is obvious to say that it is impossible to determine residual strength without destroying the specimen, which makes it very difficult to compare damage states between two specimens. Of course residual strength can be correlated with measurable manifestations of damage, but then new relations must be established between evolution of residual strength and the damage manifestation [1].

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